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FRACTAL ANALYSIS AND LONG RANGE DEPENDENCE IN S & P CNX NIFTY RETURNS

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Abstract
Fractal Analysis is an instrument that studies the repeating patterns of price over long periods of time that are created by a common driver of price that can be isolated. In assessing the fractal dimension of index returns, the retail investors, with recent innovation in financial prediction and computational power, can make use of these price movements in the stock market. The long memory describes the higher order correlation structure of a time series. The S&P CNX Nifty is a well diversified, fifty stock index, accounting for twenty two sectors of the economy. It is used for benchmarking fund portfolios, index based derivatives and index funds. The aim of this paper is to investigate the existence of long range dependence and fractal structure in the S&P CNX Nifty returns, using Rescaled Range Analysis over the period of 20 years from April 1994 to March 2014. The study found no long range dependence in the stock market.

Keywords: Fractal Analysis, Hurst Exponent, Long Range Dependence, CNX Nifty

JEL Code: C12, C15, C53, D53

I. Introduction
Fractal analysis was introduced into financial time series by Mandelbrot and Peters. Fractal analysis indicates that conventional econometric methods are inadequate for analyzing financial time series. Adequate analysis of the financial time series allows us to predict precisely the future values and risks connected with portfolios that are influenced

(Maria Bohdalova and Michal Gregus, 2010). In essence, Fractal Development is the means by which the main fundamental variable that drives a market’s price creates repeating patterns of price growth at ever higher degree over time due to repetitive market psychology (goldrunnerfractalanalysis). The Fractal Market Hypothesis (FMH) proposes that (i) The market is stable when it consists of investors,
covering a large number of investment horizons, who ensure that there is ample liquidity for traders. (ii) The information is more related to market sentiment and technical factors in the short term than in the long term - as investment horizons increase and longer term fundamental information dominates. (iii) If an event occurs that puts the validity of fundamental information in question, long-term investors either withdraw completely or invest on shorter terms (i.e. when the overall investment horizon of the market shrinks to a uniform level, the market becomes unstable). (iv) The prices react to a combination of short-term technical and long-term fundamental valuation and thus short-term price movements are likely to be more volatile than long-term trades - they are more likely to be the result of crowd behaviour. (v) If a security has no tie to the economic cycle, then there will be no long-term trend and short-term technical information will dominate.

An Index gives information about the price movements of products in the financial, commodities or any other markets. The stock market indices are meant to capture the overall behaviour of equity markets. CNX Nifty is owned and managed by India Index Services and Products Ltd. (IISL). IISL is India’s first specialised company focused upon the index as a core product. The CNX Nifty Index represents about 0.14% of the free float market capitalization of the stocks listed on NSE as on March 31, 2014. The total traded value for the last six months ending March, 2014 of all index constituents was approximately 55.67% of the traded value of all stocks on the NSE. The impact cost of the CNX Nifty for a portfolio size of Rs.50 lakhs was 0.06% for the month of March, 2014.

II. Literature Reviews

The earlier studies which analyzed the fractal structure and long range dependence in stock markets, are briefly reviewed below.

Hardayanna Rahman and Masnita Misiran (2011) investigated the long memory properties in daily returns of conventional and Islamic indices. The study found that there were significant differences, both from the conventional and the Islamic indices, in terms of their respective Hurst Value. Malhar Kale and Ferry Butar (2011) examined the distribution properties of Hurst Exponent and examined the Fractal Analysis by conducting Rescaled Range (R/S) analysis of time series. The results revealed that the estimated Hurst Exponents were also normally distributed. Mukherjee, et al., (2011) analyzed the long range dependence in stock returns of Indian Stock Market. The study found that the raw return series did not exhibit any long range dependence. Gayathri M. and Selvam M. (2011a) examined the efficiency of Fractal Market Hypothesis in the Indian Stock Market. The study found that any new information would be immediately and fully reflected in prices and stock returns of equity, suggesting that short term and long term trade followed technical information and fundamental information respectively. Gayathri M. and Selvam M. (2011b) examined the Fractal Structure in the National Stock Exchange of India in order to study the long range dependence of daily returns of Nifty in the stock market. This study showed that the time series data from Indian Stock Market were not random and the result indicated the rejection of Efficient Market Hypothesis for Indian Stock Markets during the study period. Gayathri Mahalingam, et al. (2012), in their study, found evidence of long memory in the returns from the data collected from Indian Bombay Stock Exchange. By analysing the Hurst Exponent, the study found high degree of persistence in BSE Sensex returns. Sharad Nath Battacharya and Mousumi Bhattacharya (2012) inspected the long memory properties in ten emerging stock market indices viz., BIX, CSI 300, BOVEPSA, IPSA, KLSE, KOSPI, MICEX, MXX-IPC, S&P
The result indicated that all the emerging stock market returns followed random walk. It found no evidence of long term memory in the chosen emerging stock market returns. Gayathri Mahalingam and Murugesan Selvam (2013) tested the fractal analysis in the daily index returns of CNX 500 over 2490 observations. The analysis of the study indicated that the data of index returns were found persistent. Findings of the study suggested that fractal dimension was evident in CNX 500 index during the study period. Srikanth Parthasarathy (2013) found evidence of long range dependence for the indices and the individual stocks in the Indian stock market by computing the Hurst Exponent. The study concluded that the Indian stock market witnessed significant reforms over the past two decades. Mahalingam Gayathri, et al. (2013) scrutinized the fractal dimension of company returns of ITC Limited, Reliance Industries Limited, Infosys Limited, ICICI Bank Limited and HDFC Limited, listed in the S&P CNX Nifty Index. The long memory was found for Reliance industries limited, Infosys limited, ICICI bank limited and HDFC bank returns series but not for the ITC limited returns series. Dilip Kumar and Maheswaran (2013) examined the sample indices for long range dependence. The study tested the presence of long memory in daily index returns of S&P CNX Nifty, CNX 100, S&P CNX 500, CNX Nifty Junior, Nifty Midcap 50 and CNX Smallcap from the Indian stock market. The study found that long range dependence existed in the Indian stock market of six index returns of NSE. Mahalingam Gayathri and Murugesan Selvam (2013) explored the long range dependence in the sample indices of the Indian stock market. Based on the findings, the study found that there was long term memory in the daily S&P BSE PSU index returns as the value of Hurst Exponent was greater than 0.5 and fractal dimension value was 1.4, which was less than 2.

The review of earlier studies clearly reveals the fact that there was no study exclusively covering the fractal dimension in CNX Nifty Index in the recent scenario. The effort made in this study is to fill the research gap identified in the earlier reviews on fractal dimension and long range dependence in the Indian stock market.

III. Statement of the Problem

The Efficient Market Hypothesis (EMH) states that it is impossible to beat the market because the stock market efficiency causes existing share prices to always incorporate and reflect all relevant information. According to the EMH, the stocks always trade at their fair value on stock exchanges, making it impossible for investors to either purchase undervalued stocks or sell the stocks for exaggerated prices. As such, it should be impossible to outperform the overall market through stock selection or market timing and that the only way the investors can possibly obtain higher returns is by purchasing riskier investments in the markets. There has been an ongoing argument about the validity of the EMH. Besides, there have been numerous attempts at representing a reliable price predictor. It is clear that the varying levels of awareness among investors across the globe, make it difficult to predict the prices in the stock market. Against this background, the present study examines the prediction of price through fractal analysis which may prove to be the answer to the problem of price prediction. An attempt has been made in this study to predict the prices through fractal dimension with the help of long range dependence in the Indian Stock Market.

IV. Need of the Study

The purpose of rescaled range is to provide an assessment of how the obvious variability of a time series changes with the length of the time period being considered. The
R/S analysis is superior to autocorrelation and variance analysis and also spectral analysis because it could detect long range dependence in the time series data. Hence R/S analysis was chosen for computing the fractal structure and long range dependence in the stock market.

V. Objectives of the Study

The objectives of this study were to analyze the normality, stationarity and to examine the long range dependence of CNX Nifty Returns.

VI. Hypotheses of the Study

The following null hypotheses were analyzed and tested.

- NH$_1$: There is no normality in CNX Nifty returns.
- NH$_2$: There is no stationarity in CNX Nifty returns.
- NH$_3$: There is no long range dependence in CNX Nifty returns.

VII. Data and Methodology of the Study

1. Sample Selection

NSE provides a modern, fully automated screen-based trading system, with over two lakh trading terminals, through which investors in every nook and corner of India can trade. It has played a critical role in reforming the Indian securities market and in bringing unparalleled transparency, efficiency and market integrity. NSE’s flagship index, the S&P CNX Nifty, is used extensively by investors in India and around the world to access the Indian equities market. S&P CNX Nifty is a well diversified 50 stock indices, representing all important sectors of the Indian Economy. Hence CNX Nifty was selected for this study.

2. Sources of Data and Sample Period

The present study mainly depended upon secondary data and used daily index of closing values. The closing prices of CNX Nifty were collected from the NSE official website (www.nseindia.com). The other relevant information for this study was collected from research articles and from websites. The present study covered a period of 20 years from April 1994 to March 2014.

3. Tools used for analysis

The following tools were used for the purpose of analysis of this study.

a) Descriptive Analysis

The Descriptive Analysis includes Mean, Standard Deviation, Skewness, Kurtosis and Jarque Bera.

b) Augmented Dickey Fuller Test

In statistics and econometrics, an Augmented Dickey–Fuller test (ADF) is a test for a unit root in a time series sample. The more negative it is, the stronger the rejection of the hypothesis that there is a unit root at some level of confidence. The testing procedure for the ADF test is the same as for the Dickey–Fuller test but it is applied to the model

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \cdots + \delta_p \Delta y_{t-p} + \epsilon_t \quad (1)$$

Where,

- \(\alpha\) is a constant
- \(\beta\) the coefficient on a time trend
- \(p\) the lag order of the autoregressive process.

By including lags of the order \(p\), the ADF formulation allows for higher order autoregressive processes. This means that the lag length \(p\) has to be determined while applying the test. The unit root test is carried out under the null hypothesis \(\gamma = 0\) against the alternative hypothesis of \(\gamma < 0\).

c) Rescaled Range Analysis

Hurst (1965) developed the Rescaled Range Analysis to analyze long records of
natural phenomena. Two factors are used in this analysis: firstly, the range R, which is the difference between the minimum and maximum ‘accumulated’ values or cumulative sum of X(t, tau) of the natural phenomenon at discrete integer-valued time t over a time span tau and secondly, the standard deviation S, which is estimated from the observed values X(t). Hurst found that the ratio R/S is described for a large number of natural phenomena by the following empirical relation.

\[ \frac{R}{S} = (a*N)^H \]

(2)

where, R/S = Rescaled Range; \(a\) = constant (number of intervals); \(N\) = Number of observations; \(H\) = Hurst Exponent

\[ H = \frac{\text{In}(R/S) - \text{In}(c)}{\text{In}(n)} \]

(3)

where, \(R/S\) = Rescaled Range; \(c\) = constant (number of intervals); \(n\) = time increment; \(H\) = Hurst exponent

Fractal Dimension = 2 - \(H\)

(4)

where, \(H\)=Hurst

VIII. Limitations of the Study

The following are the limitations of the present study.

- The study was confined only to CNX Nifty.
- As the study was based mainly on secondary data, it was beset with certain limitations which are bound to arise while dealing exclusively with secondary data.
- All limitations associated with various tools like Descriptive Statistics, Augmented Dickey Fuller Test and Rescaled Range Analysis, are also applicable to this study.

IX. Analysis of Normality, Stationarity and Fractal Dimension in S&P CNX Nifty Returns

For the purpose of this study, the analysis was made as follows.

1. Analysis of Normality for CNX Nifty returns
2. Analysis of Stationarity for CNX Nifty returns
3. Analysis of Long Range Dependence for CNX Nifty returns, and
4. Analysis of Fractal Dimension for CNX Nifty returns

1. Analysis of Normality for CNX Nifty returns

To test the data normality for each index returns, the study applied descriptive statistics that analyses mean value, median, minimum and maximum values, associated standard deviation, skewness, kurtosis and Jarque-Bera test. This study used a data set consisting of closing daily prices of returns of CNX Nifty. The daily data were related to a period of 20 years from April 1994 to March 2014, covering a total of 4,963 observations.

Table-1 reports the descriptive statistics of the daily returns of CNX Nifty. The mean daily return of CNX Nifty earned a positive value of 0.0005. The median daily return of CNX Nifty was 0.000 . From the above Table, the maximum and minimum values of sample index were 0.144 and -0.1224 respectively. It was observed that the standard deviation value of 0.0150 was found during the study period. It is clear that a high standard deviation was recorded which reveals high risk in the market. It is to be noted that skewness recorded at 0.0589, indicated positive sign during the study period. The excess Ourtosis (10.0841) was found for the sample index. It is to be noted that Ourtosis above the level of % is usually considered Leptokurtic. Hence there were lesser chances of extreme outcomes compared to a normal distribution. In addition to this, the analysis of Jarque-Bera statistic value of 10% 81.38 was very high, with statistically insignificant p-value at 5% or lower. Therefore, the null hypothesis (NH1), namely, There is no normality in CNX Nifty returns, is accepted.
2. Analysis of Stationarity for CNX Nifty returns

The results of Augmented Dickey Fuller Test for CNX Nifty during the period from April 1994 to March 2014, are displayed in Table-2. It is to be noted that test critical values for sample index were calculated at 1%, 5% and 10% significance levels. According to the results of the Table, the t-statistic value for sample index was -65.5313. It is significant to note that the calculated statistic value was greater than that of test critical values of -3.4315, -2.8619 and -2.5670; 1%, 5% and 10% levels of significance. Probability value of 0.0001 was recorded during the study period. Therefore, the overall analysis of the ADF Test clearly shows that there was stationarity in CNX Nifty returns during the study period. Hence the null hypothesis (NH$_2$), namely, There is no stationarity in the returns of CNX Nifty, is rejected.

3. Analysis of Long Range Dependence for CNX Nifty returns

The log (n), the log of empirical rescaled range [log (R/S)] and the log of expected rescaled range [log E(R/S)] were computed by using MATLAB algorithm, as given in Appendix-I. It is to be noted that the log R/S represents the empirical rescaled ranges while the log E(R/S) denotes the expected rescaled range analysis.

Chart-1(a) displays the log-log plot of rescaled range statistic during the study period from April 1994 to March 2014. From the Figure, the log R/S plot scale was identical with the log E(R/S) plots from the starting point 1 (April 1994) up to the point 17 (December 1994). After that, the series of log R/S moved together with log E(R/S) from January 1995 (point 18) to October 2001 (point 186). Again the plots are indistinguishable from each other from point 187 (November 2001) to point 229 (June 2003) and during point 285 (September 2005). The deviation of the plots was found between the points 245 (February 2004) and 281 (July 2005) and the points from 324 (April 2007) to 375 (April 2009). At the end of the study periods from May 2009 to March 2014 (points 376 to 497), the log R/S and log E(R/S) series moved mutually. From the overall analysis of the Chart, the series of log R/S and log E(R/S) were identical with the random walk. This clearly shows the fact that there was absence of long memory in the case of CNX Nifty during the study period from April 1994 to March 2014. Therefore, the null hypothesis (NH$_3$), namely, There is no long range dependence in the Returns of CNX Nifty, is accepted.

4. Analysis of Fractal Dimension of CNX Nifty

The results of Fractal Dimension and Hurst Exponent for CNX Nifty between April 1994 and March 2014, are depicted in Table-3. The value of Hurst Exponent was found to be 0.5163 during the study period. The value of Hurst for sample index was close to 0.5 and it would imply a random process. It is to be noted that CNX Nifty recorded the fractal dimension of 1.4837. According to the results, the sample index of CNX Nifty followed the random walk and hence the fractal dimension did not exist during the study period.

X. Conclusion and Implications

This study investigated the normality, stationarity and long memory property in the National Stock Exchange, especially in CNX Nifty returns. Non-normality and stationarity were found for CNX Nifty returns. Various studies analyzed the long range dependence in the stock market. For example, the study conducted by Hardayanna Rahman and Masnita Misiran; Malhar Kale and Ferry Butar; Mukherjee, et al., (2011) did find evidence of long term memory. Existence of long term dependence in stock markets was also found by Gayathri M. and Selvam M. (2011b),

Fractal Analysis and Long Range Dependence in S & P CNX Nifty Returns
Gayathri Mahalingam, et al; SharadNath Battacharya and Mousumi Bhattacharya (2012), Gayathri Mahalingam and Murugesan Selvam; Srikanth Parthasarathy; Mahalingam Gayathri, et al. (2013). These findings are not consistent with the present study. The Hurst Exponent of sample index of present study shows no long-term dependence during the study period. It is found that the daily returns of CNX Nifty did not confirm the long range dependency. In other words, it was consistent with the random walk behaviour. The results suggested that the data of CNX Nifty were not influenced by the past data. The participants in the National Stock Exchange market may consider the short term movements while determining the dynamics of their investment assets.

XI. Scope for further study

A study, with similar objectives, could be made with reference to other types of indices like Asian stock market indices, world stock market indices, etc. Future study may evaluate the long range dependence on squared returns, absolute returns, weekly returns, monthly returns and one minute interval. A study, with similar objectives, could be made from time to time. A study may be conducted to investigate the long range dependence across the individual stock prices.

References


APPENDIX – I
MATLAB ALGORITHM FOR COMPUTING THE RESCALED RANGES

```matlab
function [out]=RSA(x)
%Syntax: [out]=RSA(x)
% Output format: 1st column=N; 2nd column=logarithm of n [log (n)]; 3rd column = logarithm of empirical rescaled ranges [log (R/S)]; 4th column = logarithm of % expected rescaled ranges [log E(R/S)] (random process)
% This function performs the classical rescaled range analysis to a time
% series. The algorithm computes the rescaled ranges (R/S), the expected rescaled % ranges of a random process E(R/S) according Anis and Lloyd (1976).
% This algorithm is based on earlier work by Thomas A. Thiele (2006).
if nargin<1 || isempty(x)
    error(’Please enter a time series’);
else
    if min(size(x))<1
        error(’Invalid time series’);
    end
    x=x(:);
    N=length(x);
end
n=10:10:4960;
for i=1:length(n)
    % Calculate the subperiods
    a=floor(N/n(i));
    % Make the subperiods’ matrixes
    X=reshape(x(1:a*n(i)),n(i),a);
    % Estimate the mean of each subperiod
    ave=mean(X);
    % Remove the mean from each subperiod
    cumdev=X-ones(n(i),1)*ave;
    % Estimate the cumulative deviation from the mean
    cumdev=cumsum(cumdev);
    % Estimate the standard deviation
    stdev=std(X);
    % Estimate the rescaled range
    rs=(max(cumdev)-min(cumdev))./stdev;
    % Take the logarithm of the mean rescaled ranges
    logRS(i,1)=log10(mean(rs));
    % The estimation of log(E(R/S))
    j=1:n(i)-1;
    s=sqrt((n(i)-j)./j);
    s=sum(s);
    % Using the formula provided by Anis and Lloyd (1976)
    logERS(i,1)=log10((gamma(0.3*(n(i)-1))/(sqrt(pi)*gamma(.3*n(i))))*s);
    logn(i,1)=log10(n(i));
    nrange(i,1)=n(i);
    out=[nrange,logn,logRS,logERS];
end
end
```

Websites
http://www.goldrunnerfractalanalysis.com
http://www.nseindia.com
http://www.wikipedia.org/CNX_Nifty
http://www.yahoofinance.com
Table-1: Results of Descriptive Statistics of CNX Nifty during the period from April 1994 to March 2014

<table>
<thead>
<tr>
<th>Descriptive Statistics</th>
<th>CNX NIFTY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0005</td>
</tr>
<tr>
<td>Median</td>
<td>0.0007</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.1774</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.1224</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.0160</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.0689</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>10.0841</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>10381.58</td>
</tr>
<tr>
<td>Observations</td>
<td>4963</td>
</tr>
</tbody>
</table>

Source: www.nseindia.com and computed using E-Views (Version 7)

Table-2: Results of Augmented Dickey Fuller Test for CNX Nifty during the period April 1994 to March 2014

<table>
<thead>
<tr>
<th>CNX Nifty (1994-2014)</th>
<th>t statistic</th>
<th>Probability*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>-65.5313</td>
<td>0.0001*</td>
</tr>
<tr>
<td>Test Critical Values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1% level</td>
<td>-3.4315</td>
<td></td>
</tr>
<tr>
<td>5% level</td>
<td>-2.8619</td>
<td></td>
</tr>
<tr>
<td>10% level</td>
<td>-2.5670</td>
<td></td>
</tr>
</tbody>
</table>

# Significant at 1% level of significance

Source: www.nseindia.com and computed using E-Views (Version 7)

Table-3: Analysis of Fractal Dimension of CNX Nifty Returns

<table>
<thead>
<tr>
<th>Sample Index</th>
<th>Hurst Exponent</th>
<th>Fractal Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNX Nifty</td>
<td>0.5163</td>
<td>1.4837</td>
</tr>
</tbody>
</table>

Chart-1: R/S Analysis for CNX Nifty Returns during the period from April 1994 to March 2014